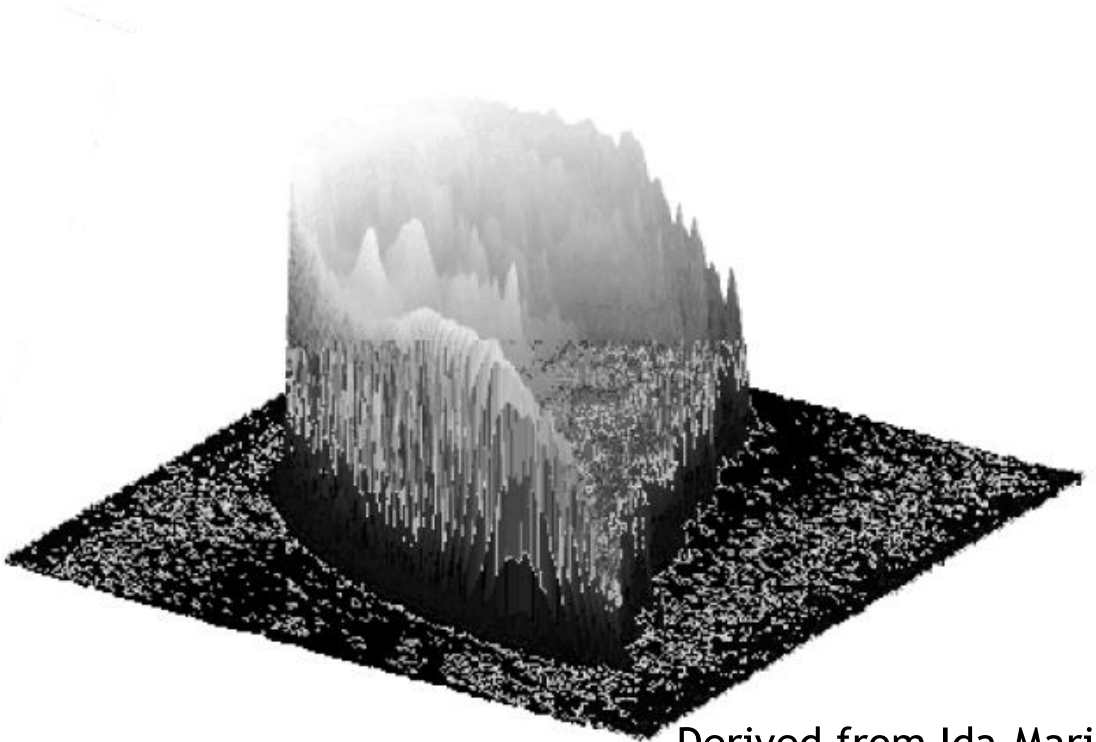
Mathematical Morphology second part: 3D

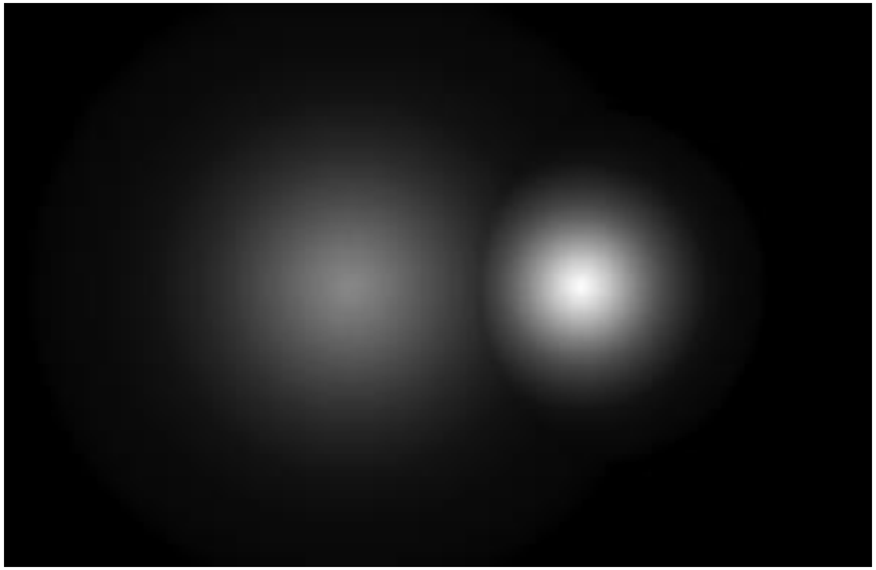
Extension to Grayscale Image

- A 2D grayscale image is treated as a 3D solid in space – a landscape – whose height above the surface at a point is proportional to the brightness of the corresponding pixel.

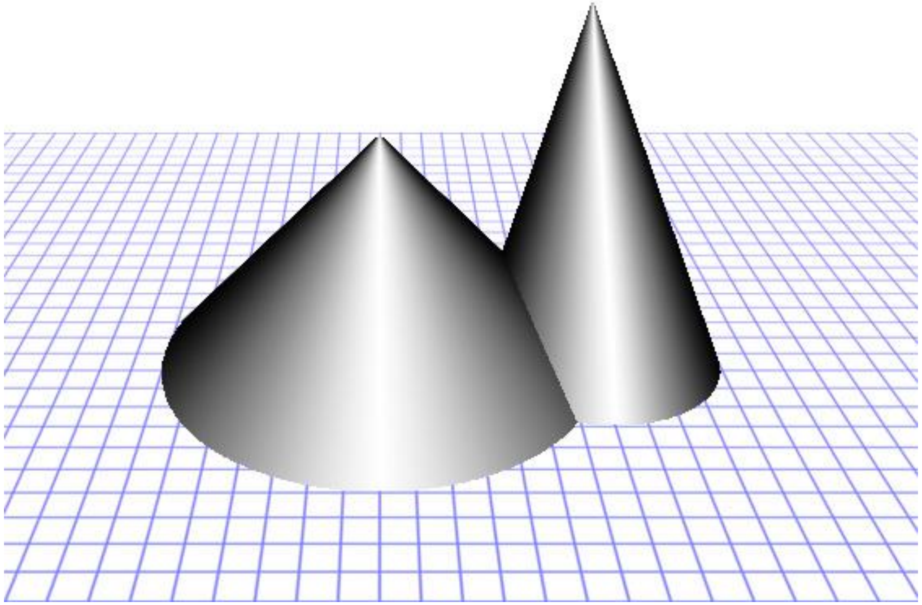


Derived from Ida-Maria Sintorn

Representation of Grayscale Images

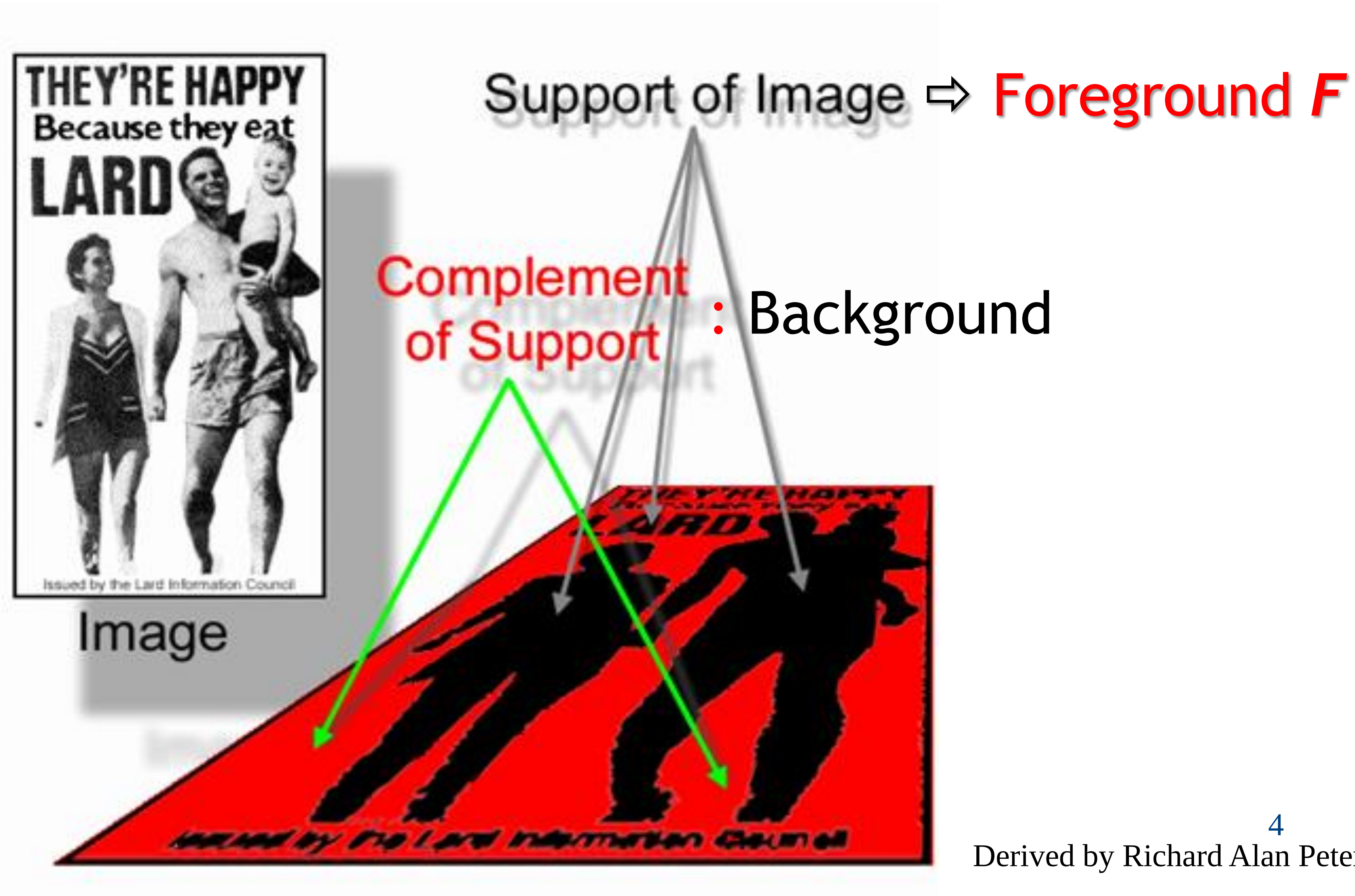


image



landscape

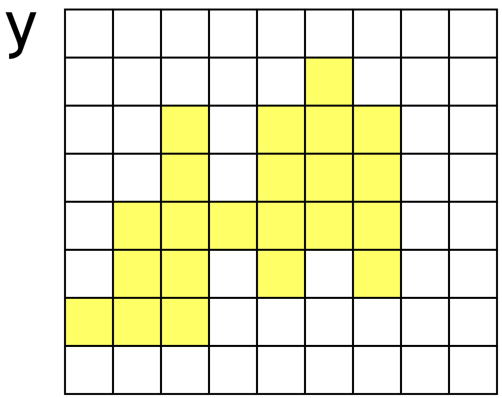
Support of an Image



Derived by Richard Alan Peters II

Umbra

- This part of mathematical morphology is an extension to multidimensional 'images' in particular to grey level and color images
- $A \subseteq E^n$, $F \subseteq E^{n-1}$ (support of A), $x \in F$ (support element), $y \in E$ (intensity)
- Top of a set A (for n=2):
  $T[A](x) = \max \{ y \mid (x, y) \in A \}$
- Umbra of f:
  $U[f] = \{ (x, y) \in A \mid y \leq f(x) \}$



Set A

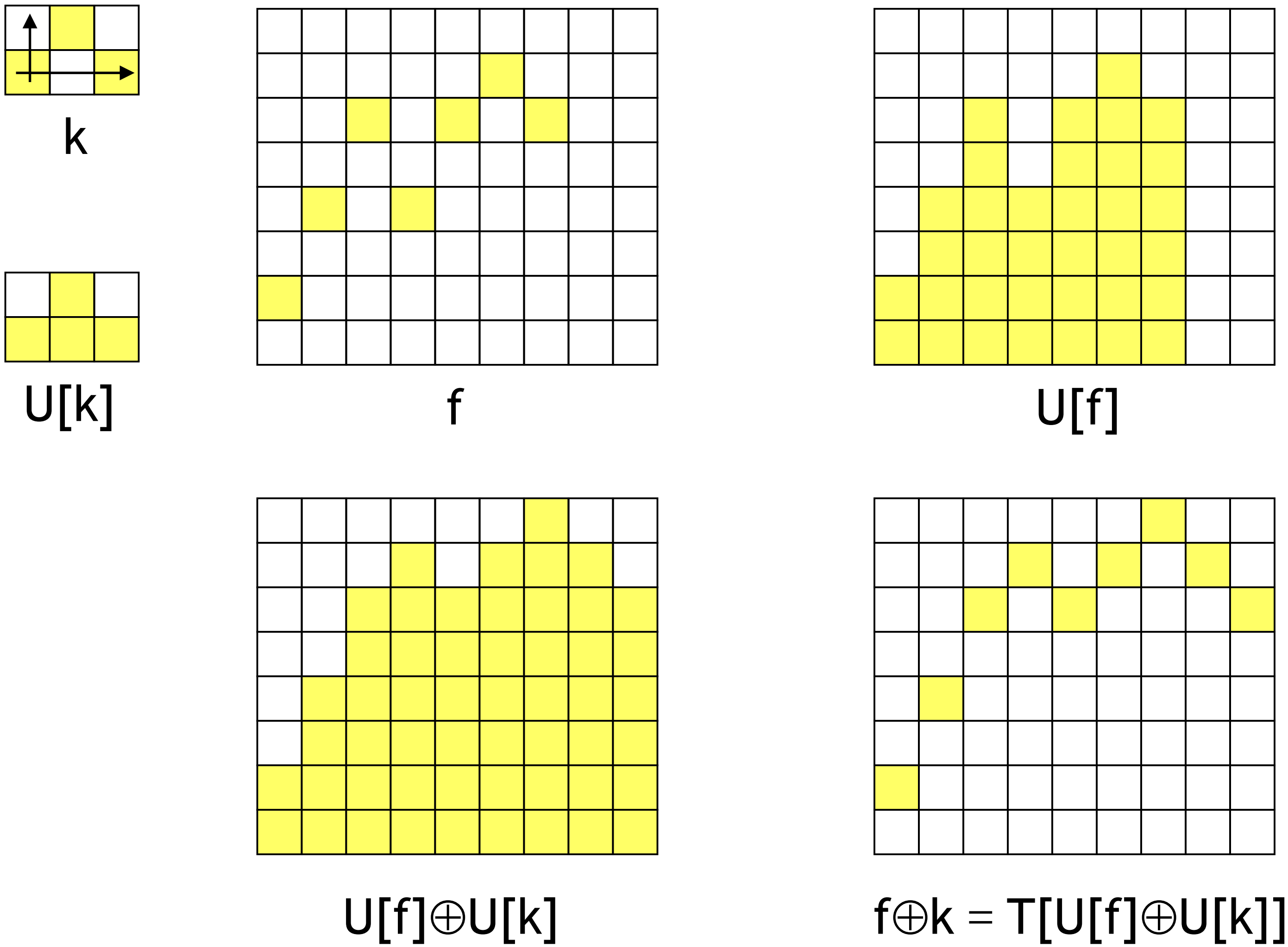
Umbra - Proprieties

- $T[A] \subseteq A \subseteq U[A] \subseteq E^n$
- $U[U[A]] \equiv U[A]$

Dilation for grey level images

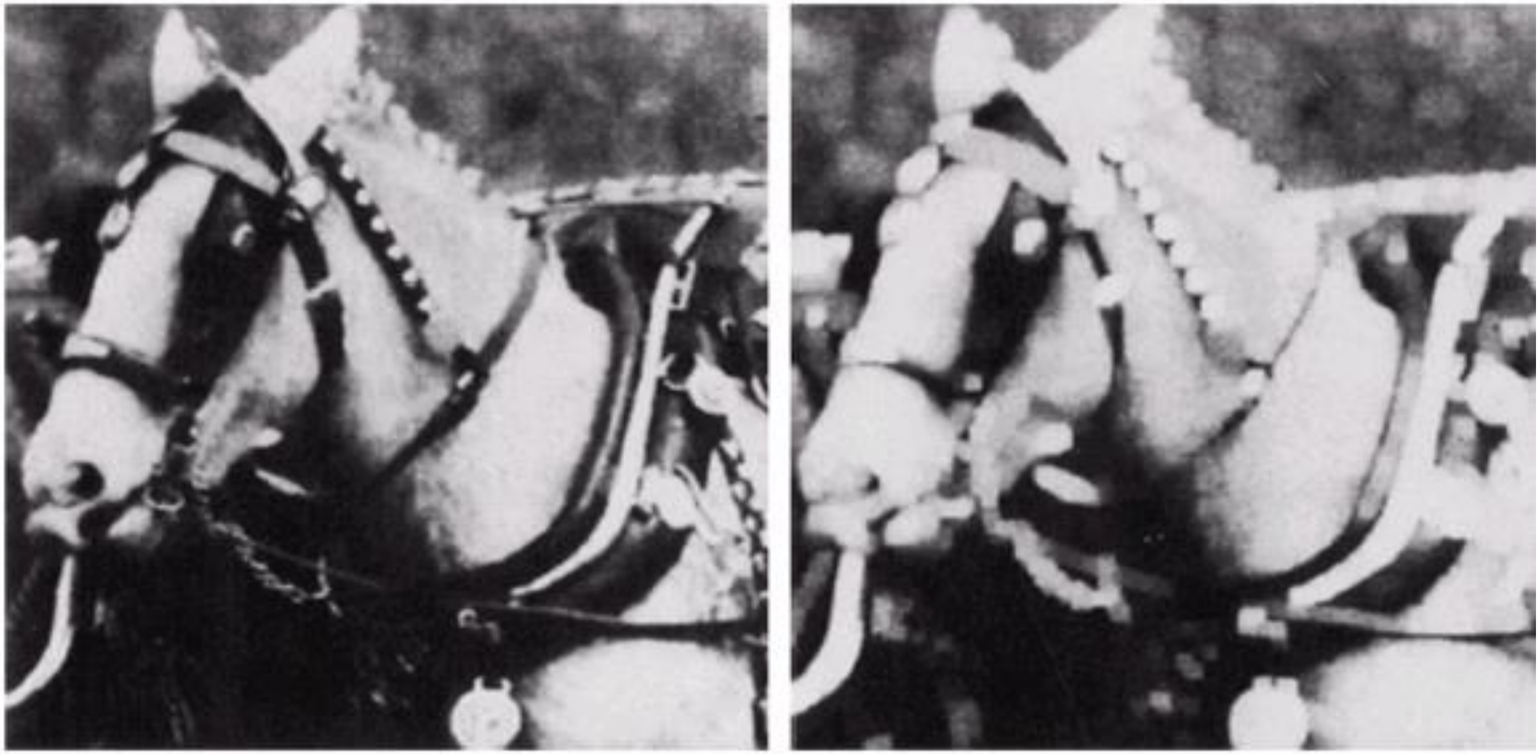
- $F,K \subseteq E^{n-1}$
- The dilation of image f and structural element k can be defined as:
  $f \oplus k = T\{U[f] \oplus U[k]\}$
- Tends to brighten the image, remove dark regions

Dilation - Example



Dilation - Example

- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels



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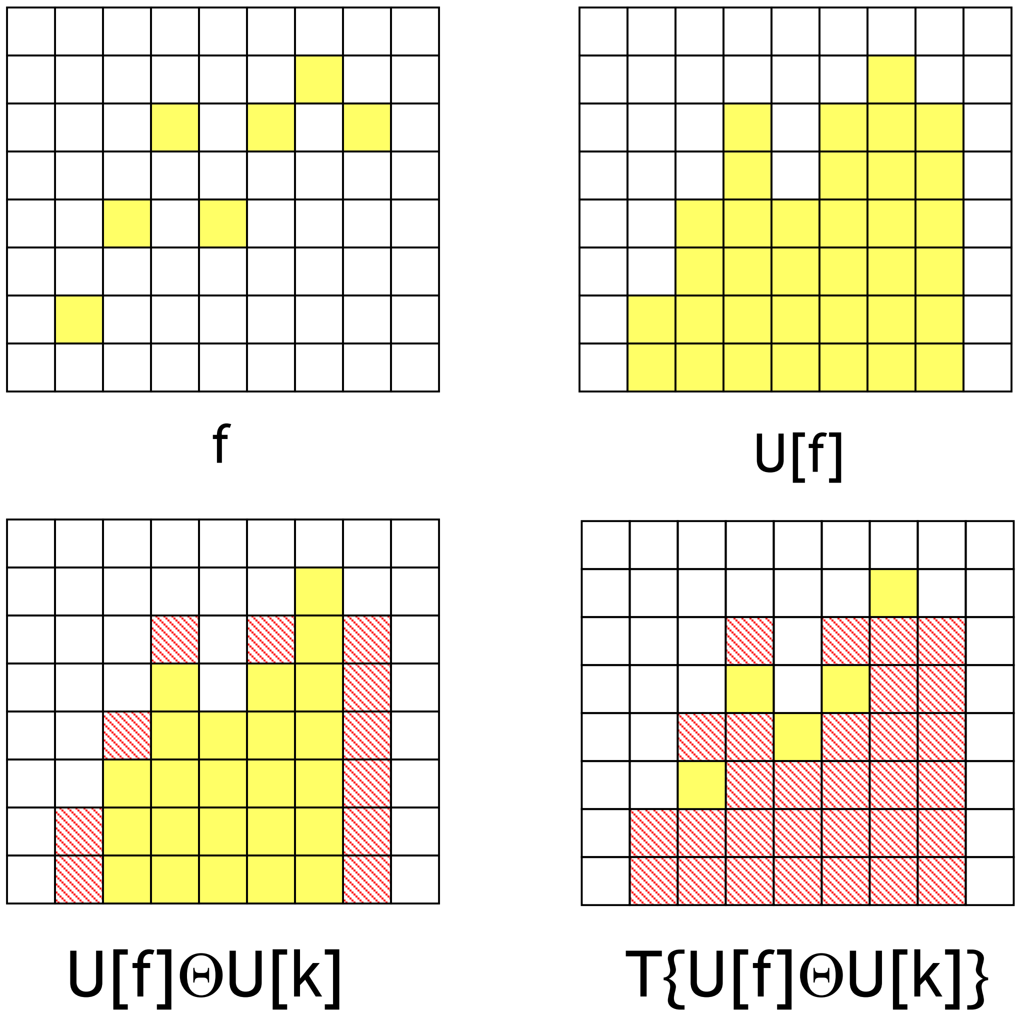
Grey scale erosion

- $F, K \subseteq E^{n-1}$
- The erosion of image f and structural element k can be defined as:
  $f \ominus k = T\{U[f] \ominus U[k]\}$
- Tends to darken the image, remove bright regions
- From the computational view point this operation is equivalent to a convolution

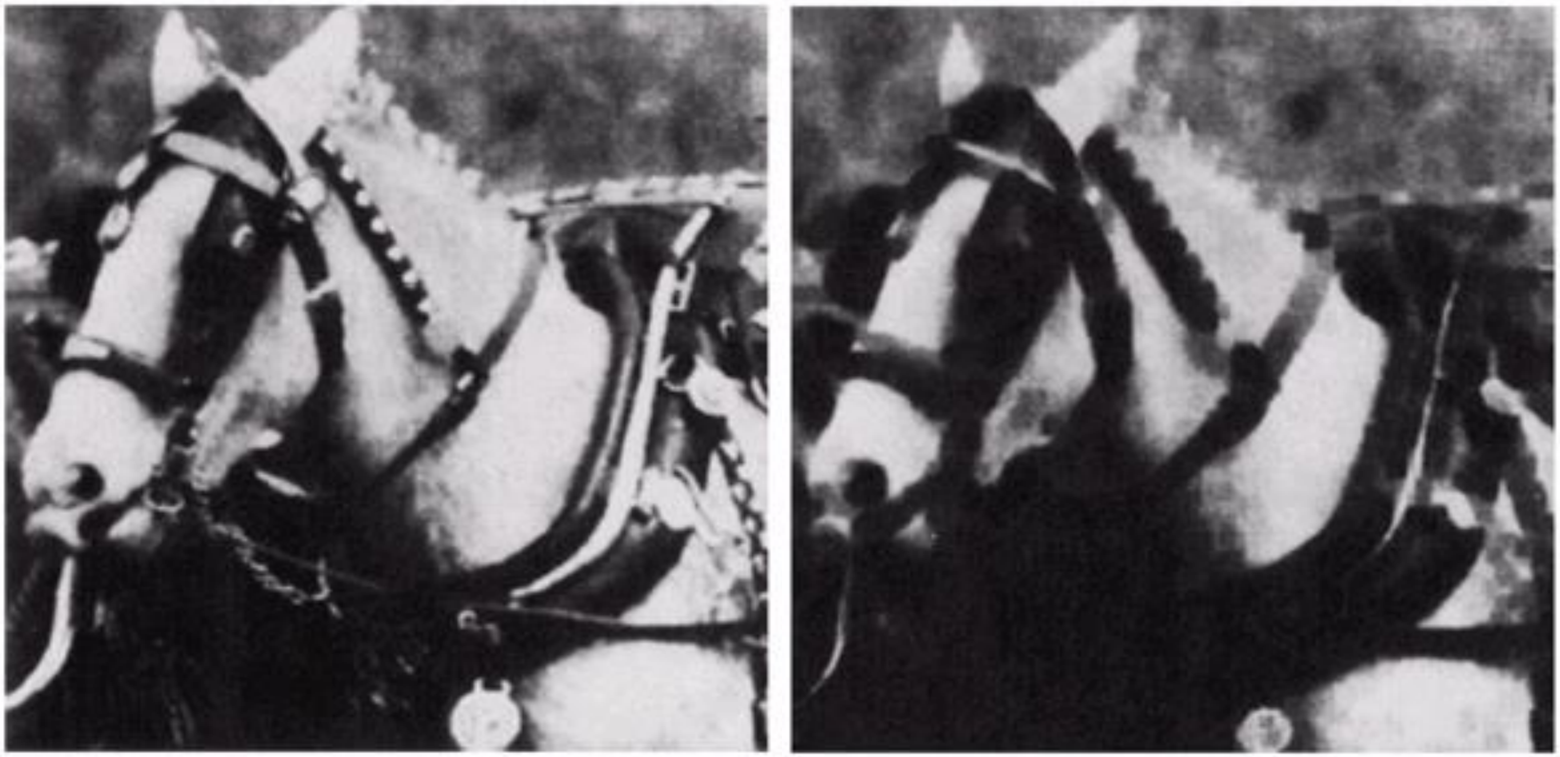
Erosion - Example

U[k]



Erosion - Example

- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels



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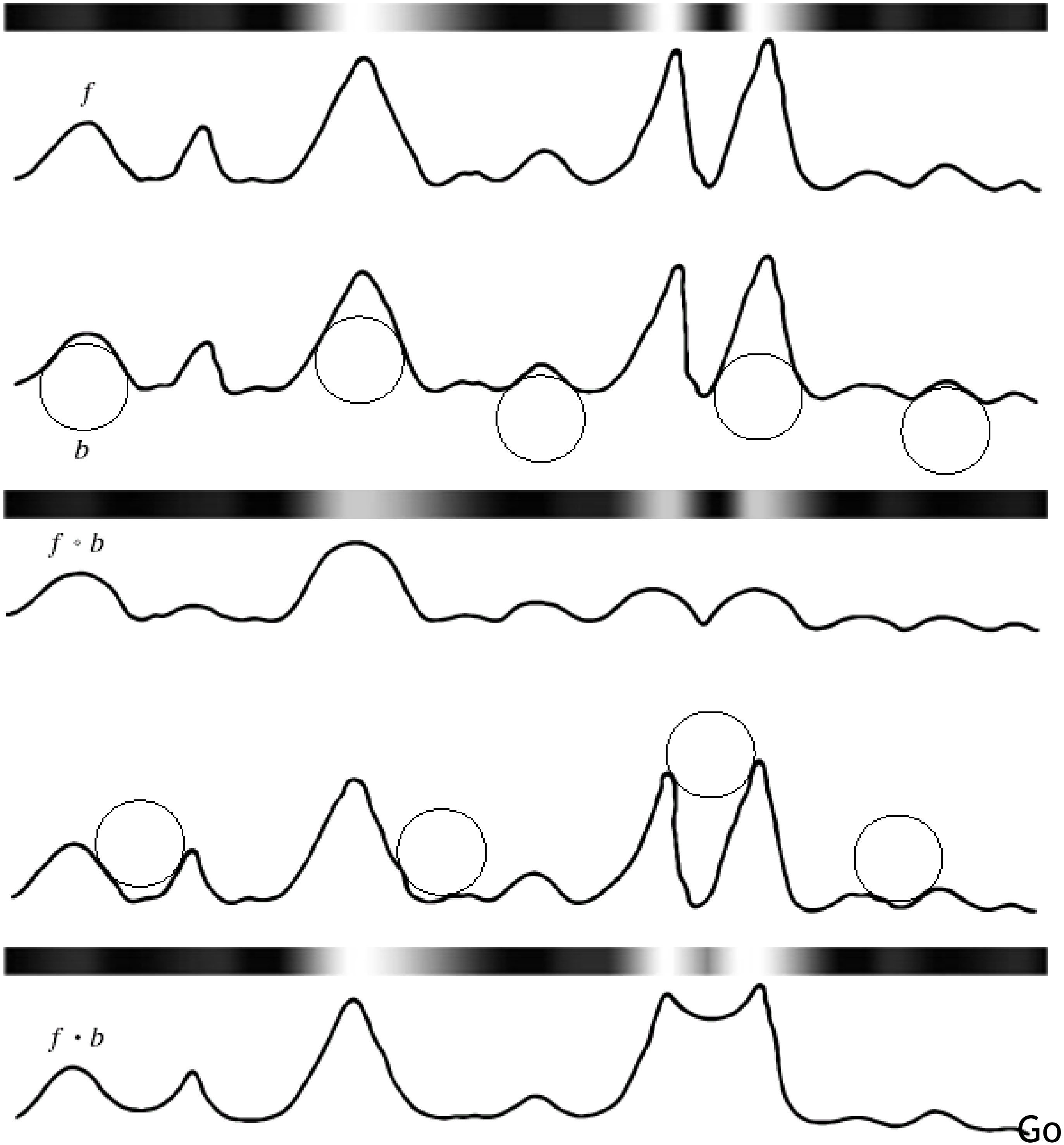
Opening and Closing

- **Opening** and **closing** of an image *f(x,y)* by a structuring element *b(x,y)* have the same form as their binary counterpart:

$$f \circ b = (f \ominus b) \oplus b \qquad\qquad f \bullet b = (f \oplus b) \ominus b$$

- Geometric interpretation:
- View the image as a 3-D surface map, and suppose we have a spherical structuring element.
  - **Opening**: roll the sphere against the *underside* of the surface, and take the *highest* points reached by any part of the sphere. Opening suppresses bright details smaller than the specified SE.
  - **Closing**: roll the sphere *on top* of the surface, and take the *lowest* points reached by any part of the sphere. Closing suppresses dark details smaller than the specified SE.
- Opening and closing are used often in combination as morphological filters for image smoothing and noise removal.



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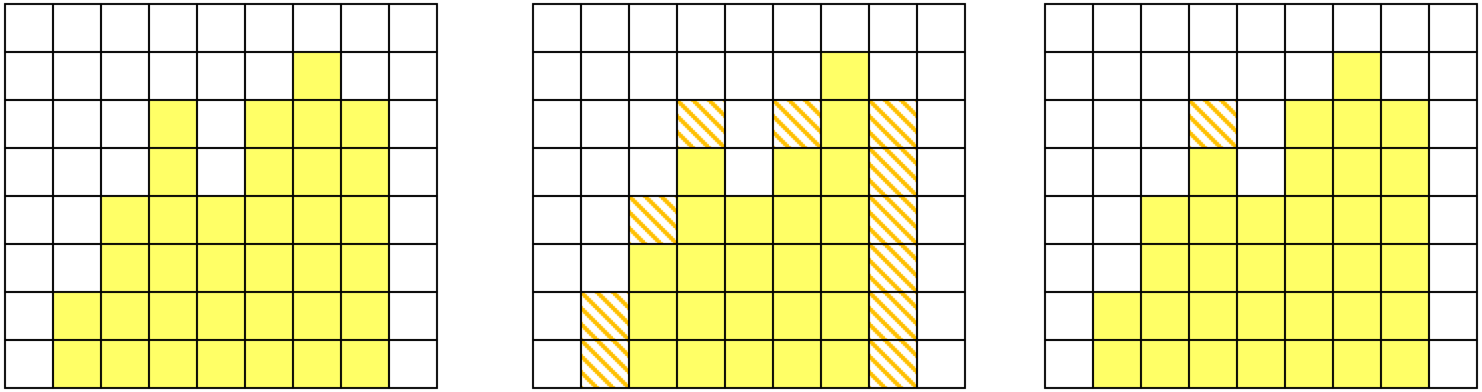
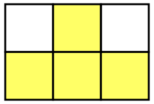
Opening - Example



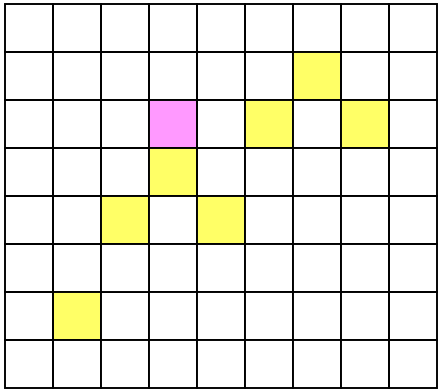
U[A]

U[A]ΘU[k]

U[A]ΘU[k]⊕U[k]



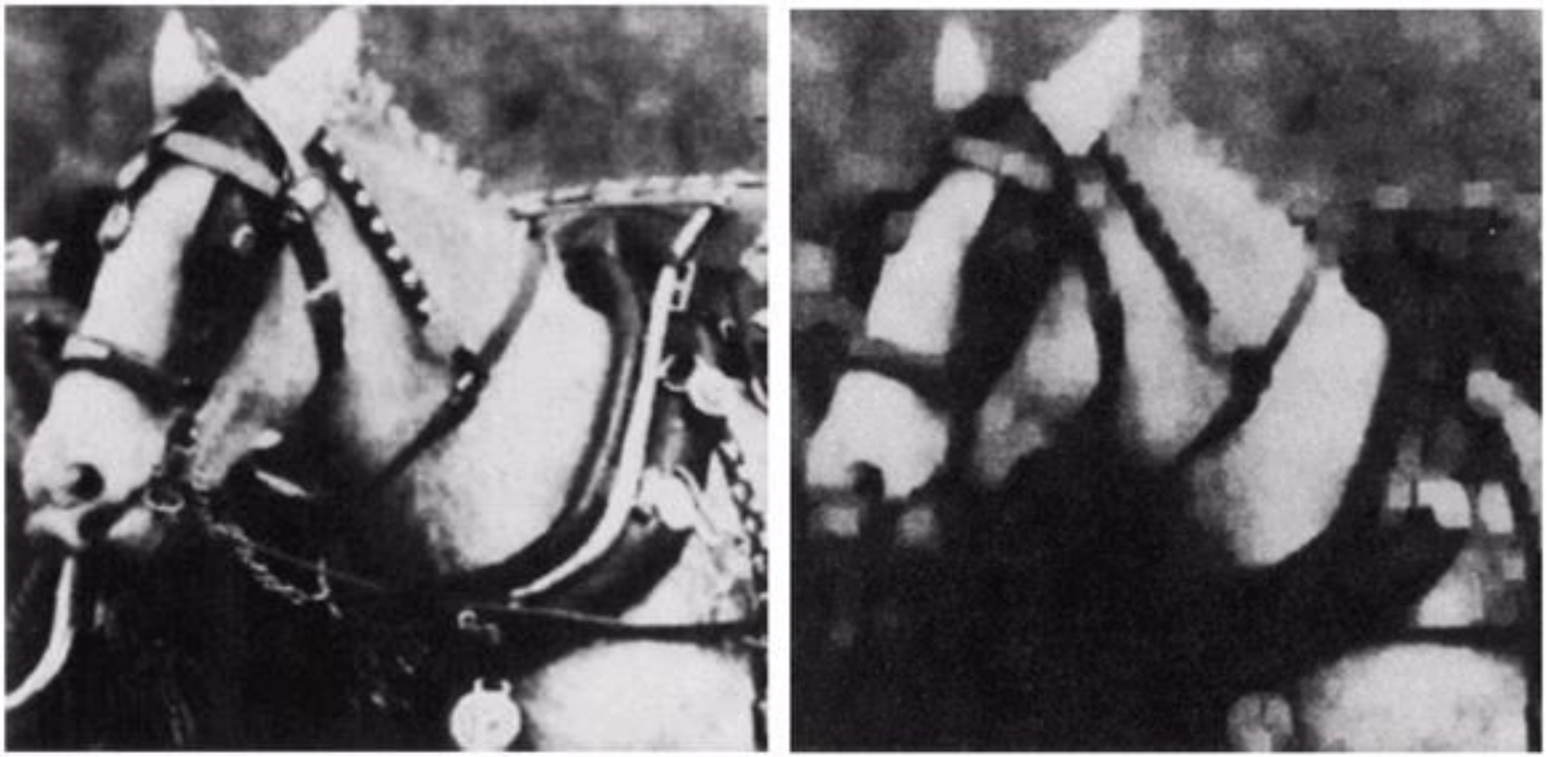
U[k]



$A \circ k$

Opening - Example

- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels
- Note the decreased size of the small bright details with no appreciable effect on the darker details



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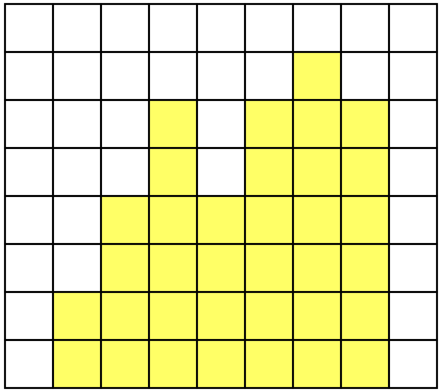
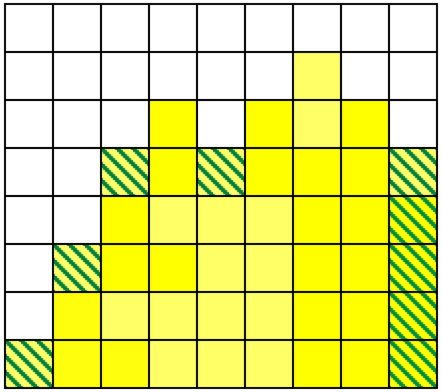
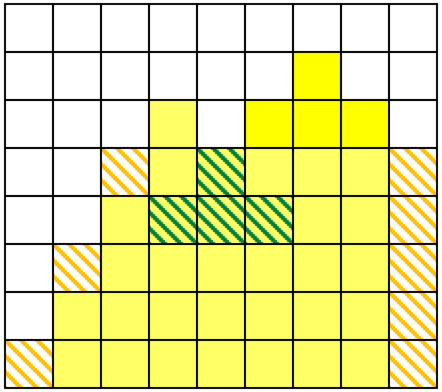
Closing - Example



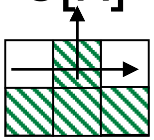
U[A]



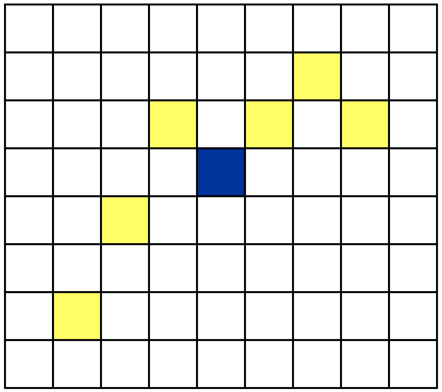
$U[A] \oplus U[k]$



$[U[A] \oplus U[k]] \ominus U[k]$



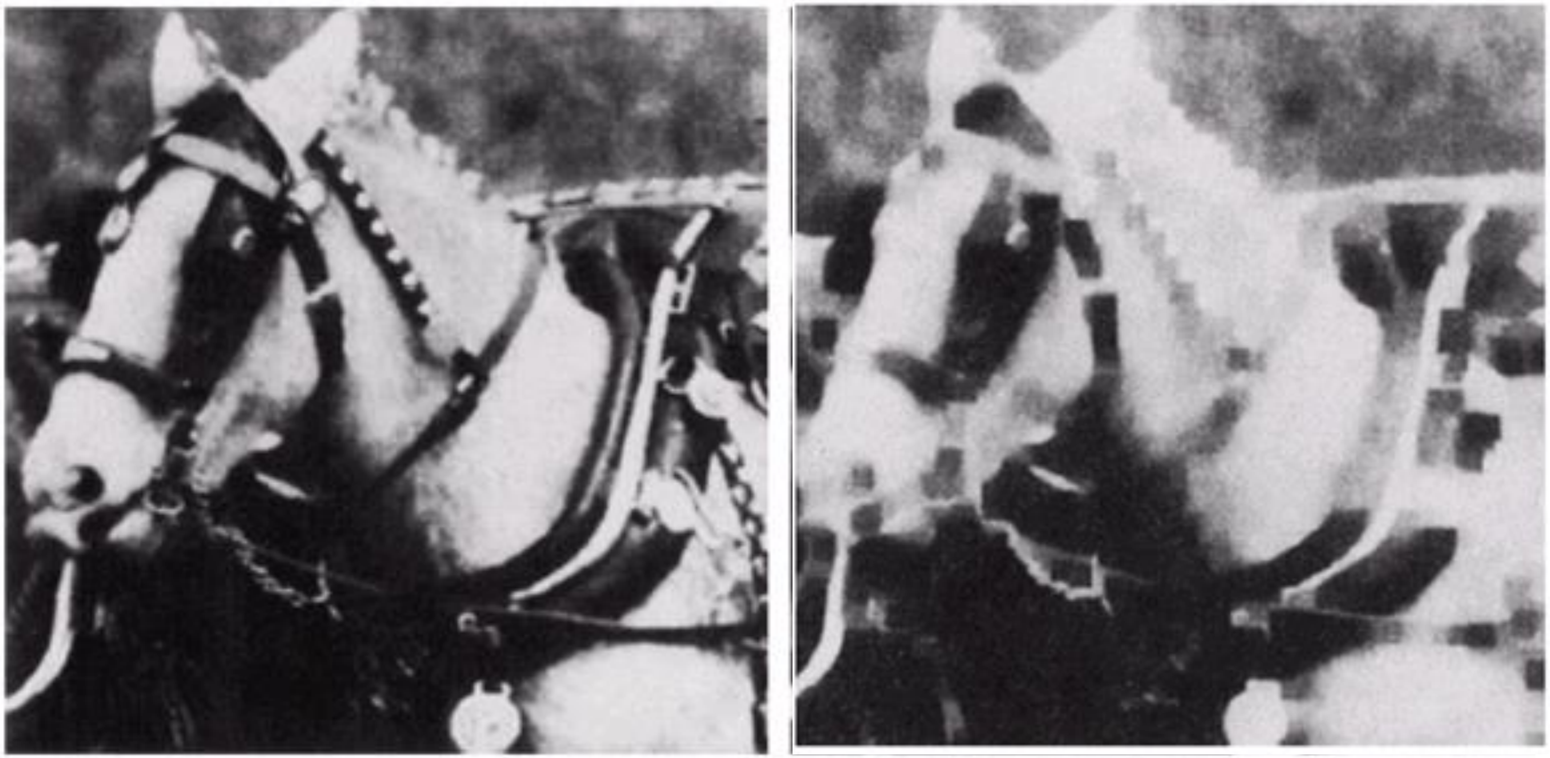
U[k]



$A \bullet k$

Closing - Example

- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels
- Note the decreased size of the small darker details with no appreciable effect on the bright details



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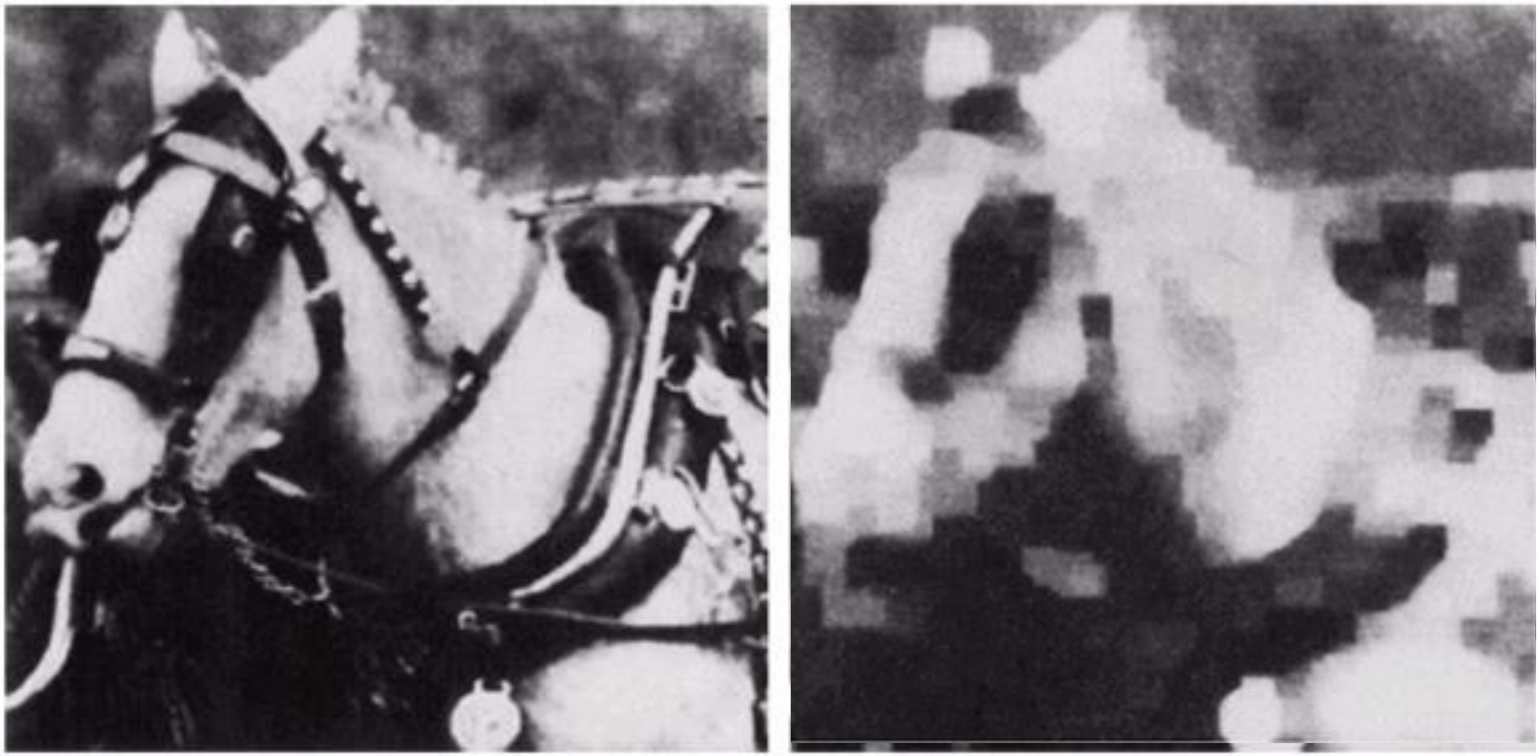
Morphological smoothing - Example

- Opening followed by closing
- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels



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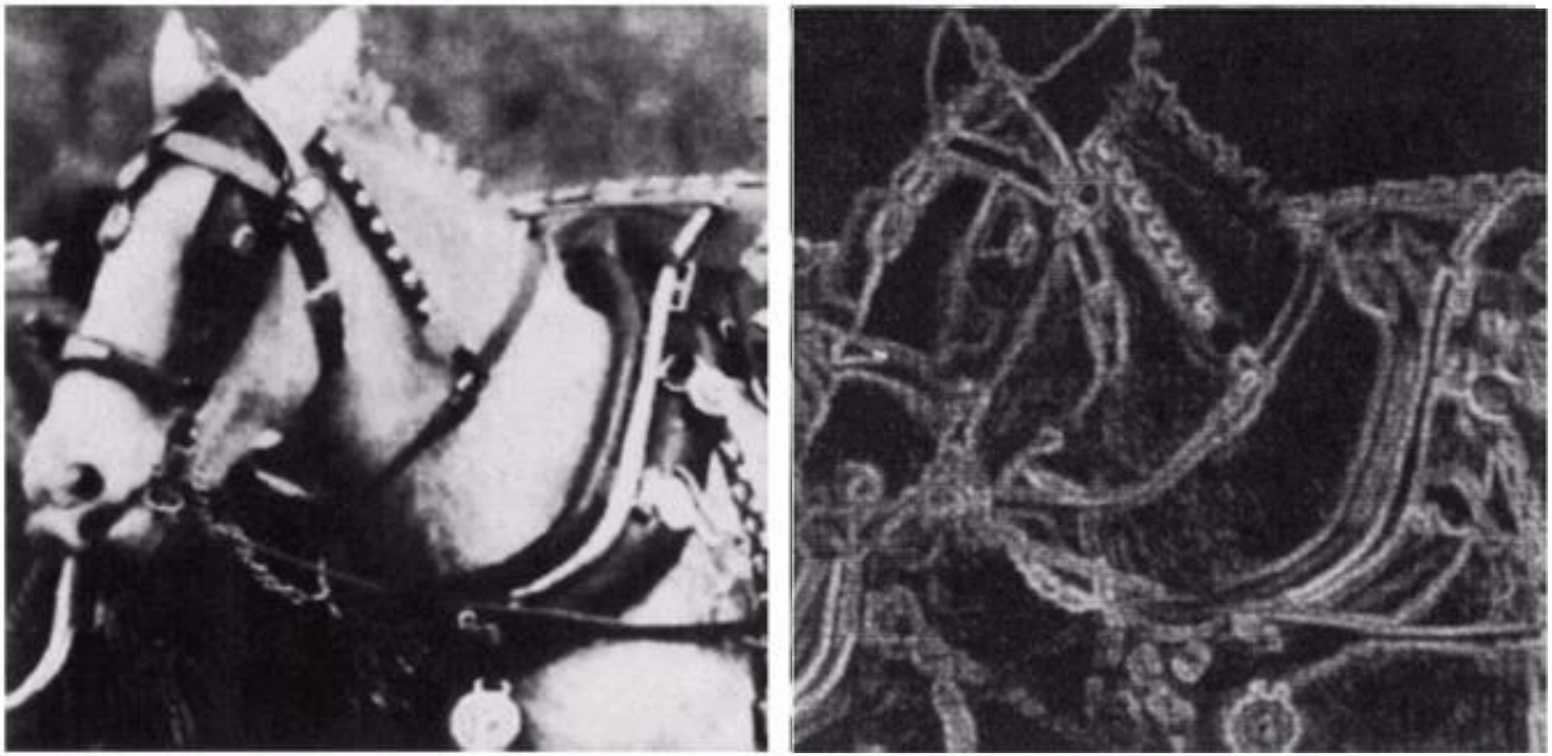
Morphological gradient - Example

- Difference between dilation and erosion $g = (f \oplus b) - (f \ominus b)$
- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels
- The edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a “derivative-like” (gradient) effect.



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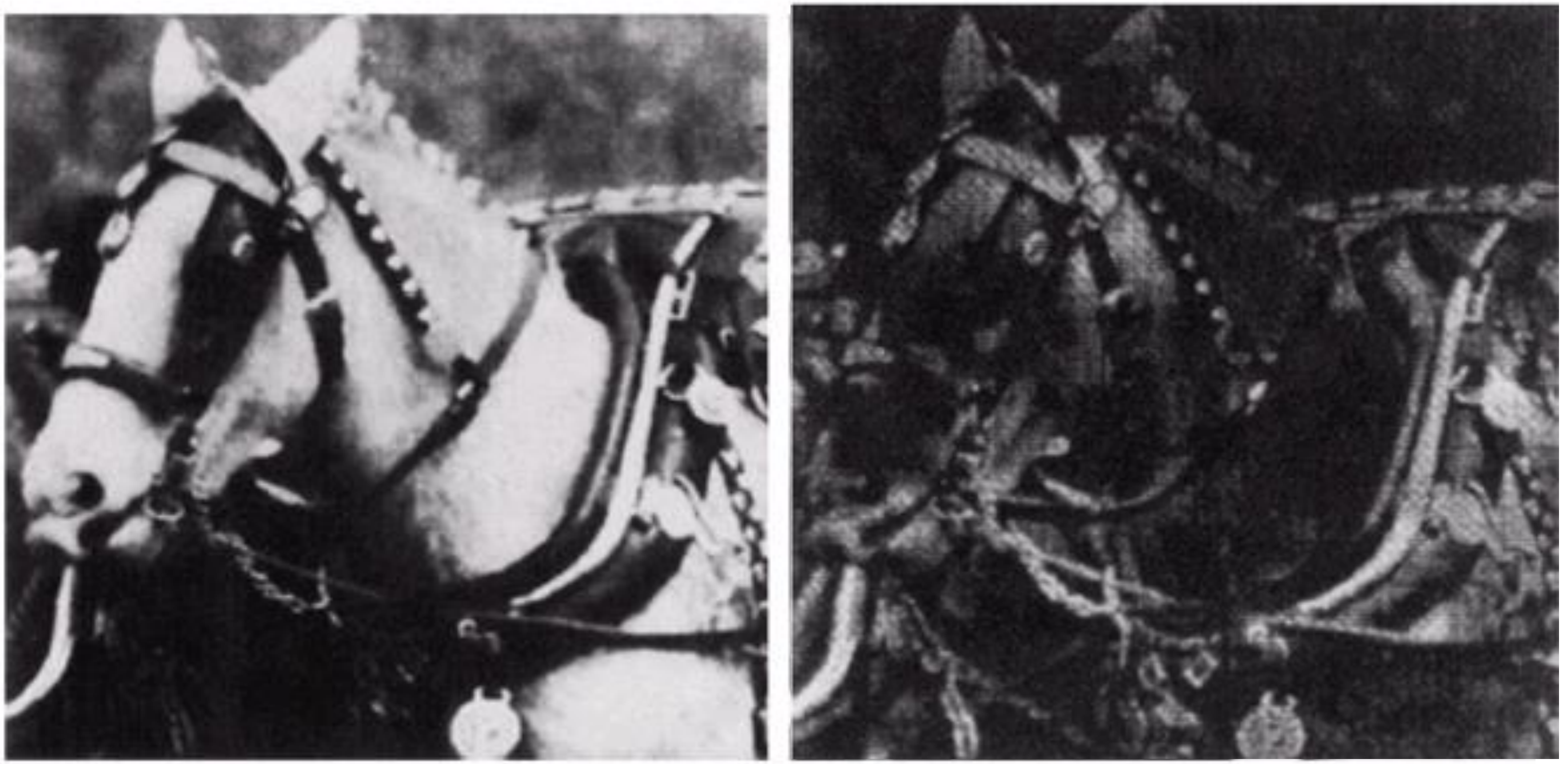
Top-hat transformation - Example

- Difference between original and opening $g = f - (f \circ b)$
- Structuring element: “flat-top”, a parallelepiped with unit height and size 5x5 pixels



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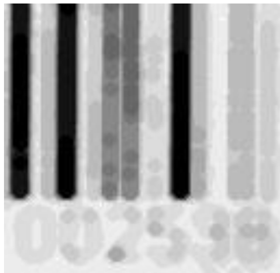
Opening vs Closing on Gray Value Images

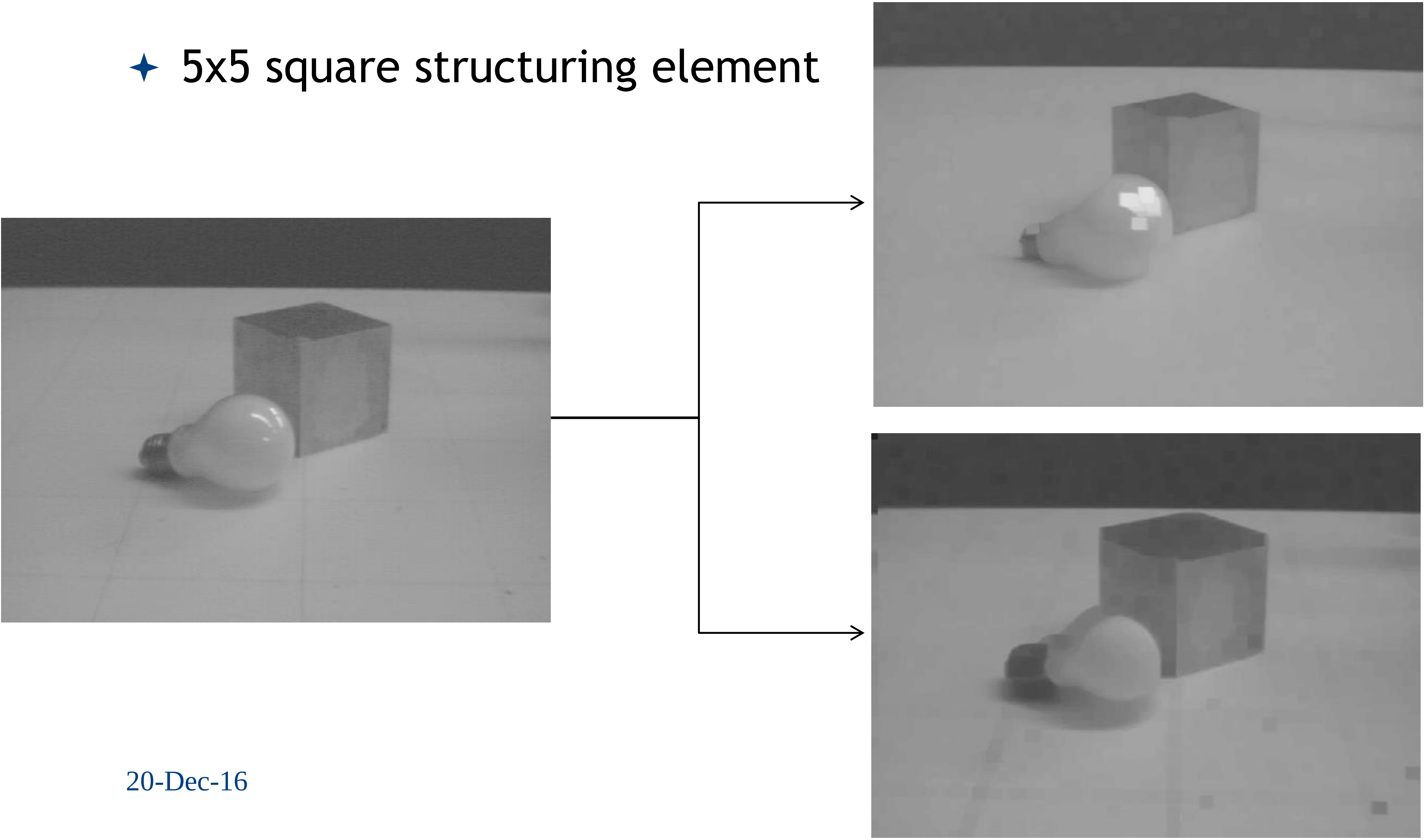
- Disk with radius 3 as structure element
- Opening: all the thin white bands have disappeared, only the broad one remains

- Closing: all the valleys where the structure element does not fit have been filled, only the three broad black bands remain



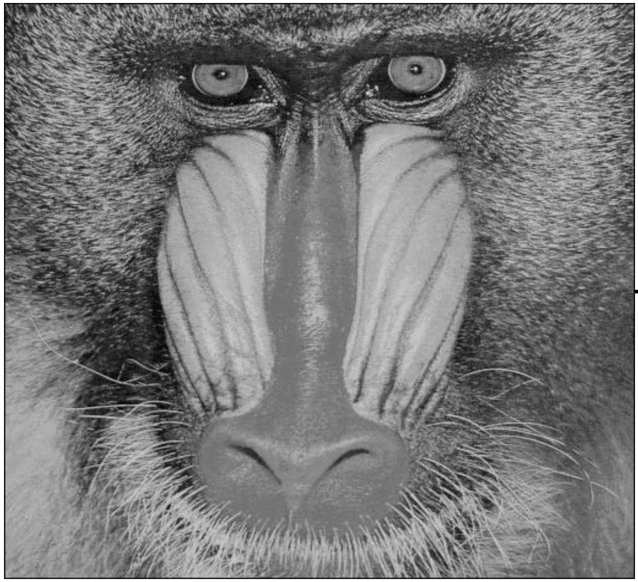
Opening vs Closing on Gray Value Images

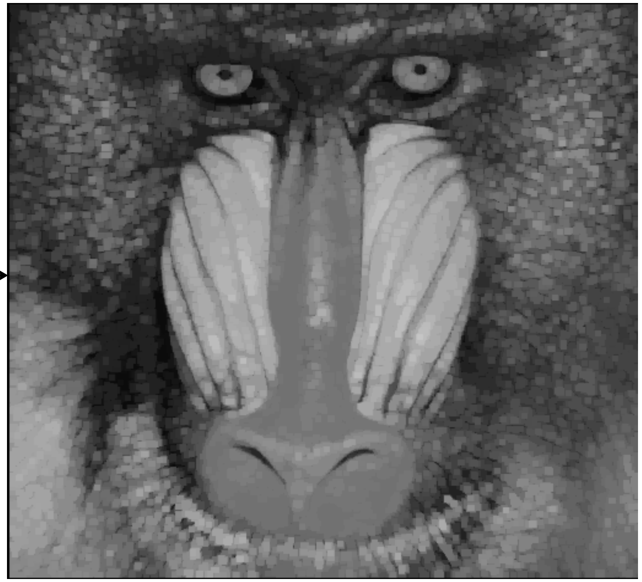
- 5x5 square structuring element

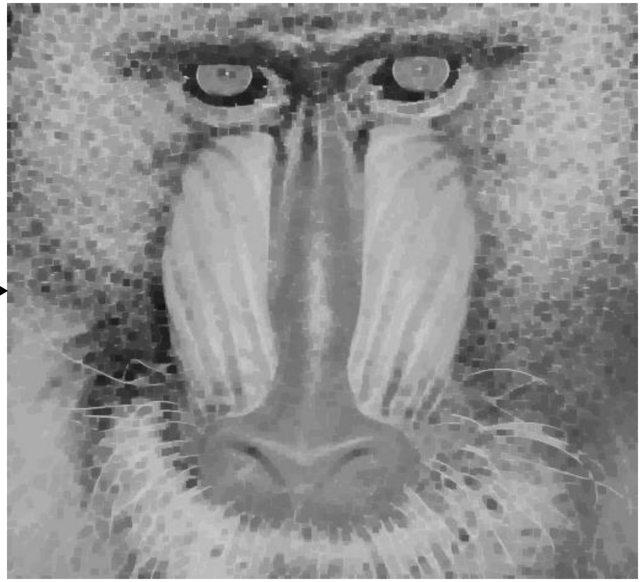


Opening vs Closing on Gray Value Images

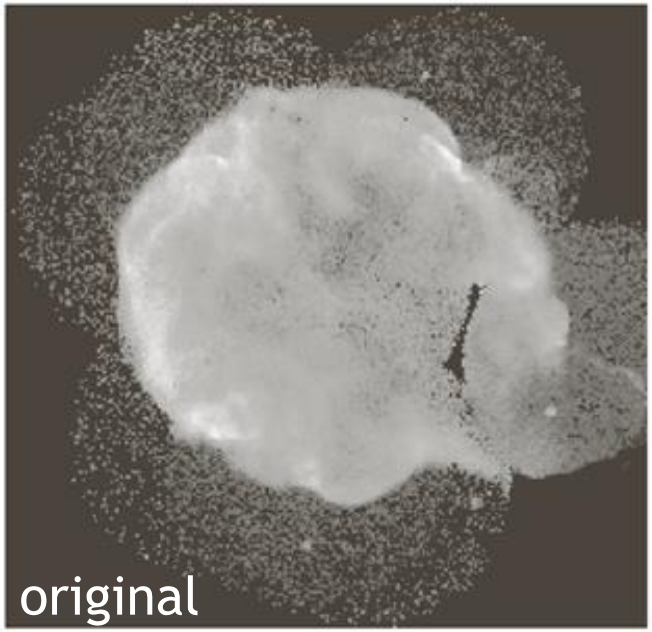
- 5x5 square structuring element







Morphological Smoothing: opening and closing



Cignus Loop supernova, taken by X-ray band by NASA Hubble Telescope

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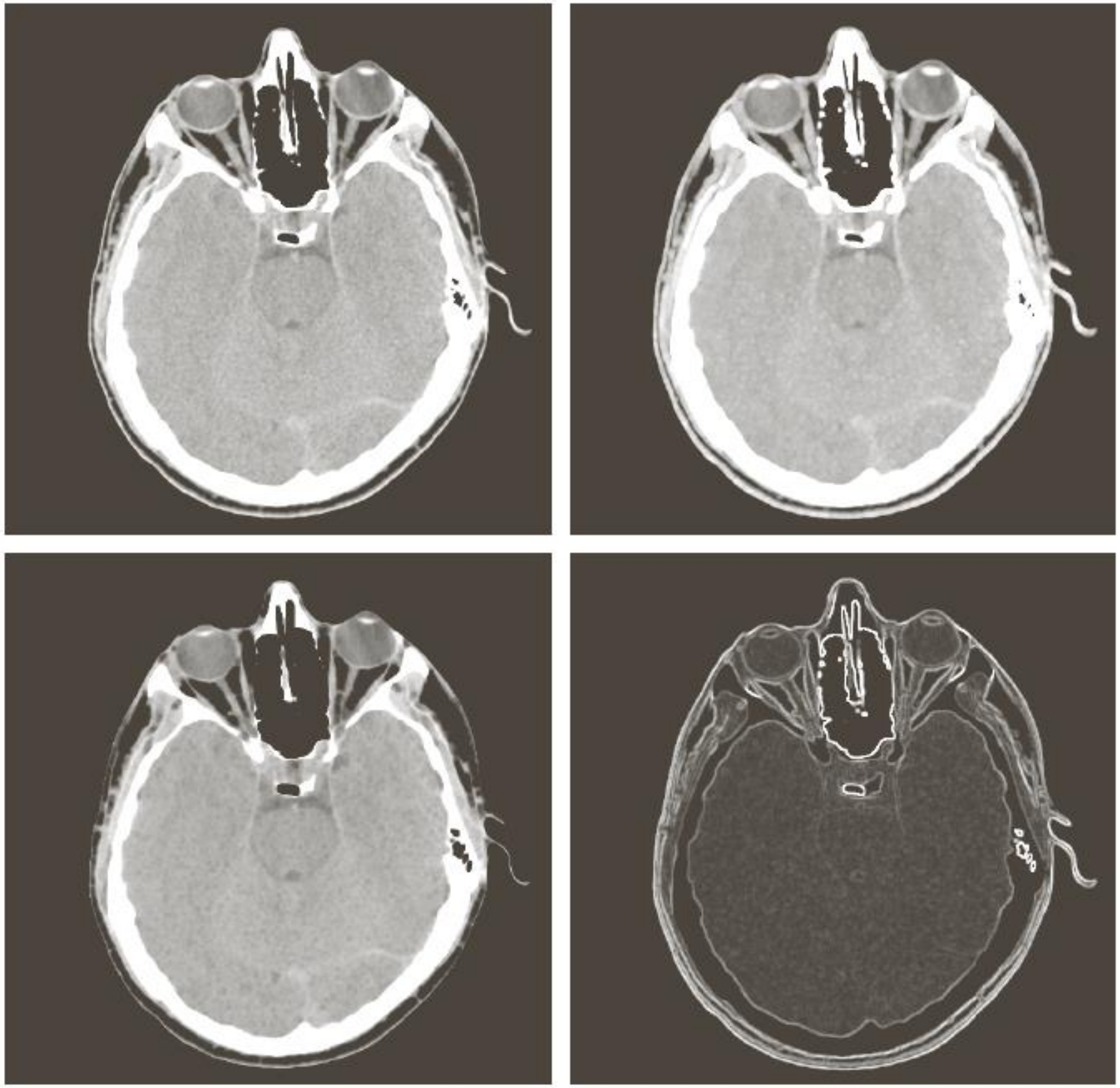
Morphological Gradient



a	b
c	d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

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Top-hat and Bottom-hat Transformations

- The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

- The bottom-hat transformation of a grayscale image f is defined as its closing minus f:

$$B_{hat}(f) = (f \bullet b) - f$$

- One of the principal applications of these transformations is in removing objects from an image by using structuring element in the opening or closing operation

Example of Using Top-hat Transformation in Segmentation

a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

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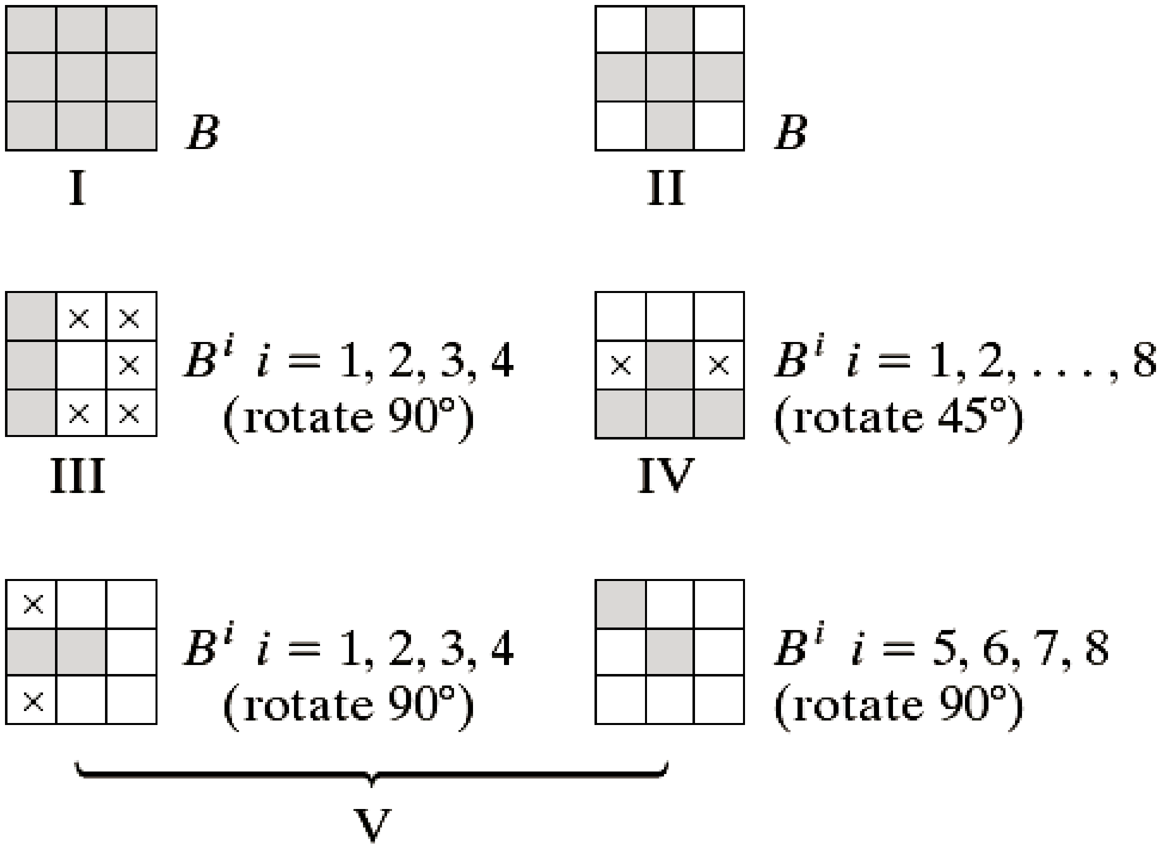
Summary



- X's indicate «don't care» values

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Summary

Operation	Equation	Comments
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(Continued)

Gonzales-Woods

Summary

Operation	Equation	Comments
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c;$ $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A;$ $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)

(Continued)

Gonzales-Woods

Summary

Operation	Equation	Comments
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A;$ $i = 1, 2, 3, 4;$ $k = 1, 2, 3, \ldots;$ $X_0^i = A;$ and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.

(Continued)

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Summary

Operation	Equation	Comments
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I

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Operation	Description
`bothat`	"Bottom-hat" operation using a 3 × 3 structuring element; use `imbothat` (see Section 9.6.2) for other structuring elements.
`bridge`	Connect pixels separated by single-pixel gaps.
`clean`	Remove isolated foreground pixels.
`close`	Closing using a 3 × 3 structuring element; use `imclose` for other structuring elements.
`diag`	Fill in around diagonally connected foreground pixels.
`dilate`	Dilation using a 3 × 3 structuring element; use `imdilate` for other structuring elements.
`erode`	Erosion using a 3 × 3 structuring element; use `imerode` for other structuring elements.
`fill`	Fill in single-pixel "holes" (background pixels surrounded by foreground pixels); use `imfill` (see Section 11.1.2) to fill in larger holes.
`hbreak`	Remove H-connected foreground pixels.
`majority`	Make pixel p a foreground pixel if at least five pixels in $N_8(p)$ (see Section 9.4) are foreground pixels; otherwise make p a background pixel.
`open`	Opening using a 3 × 3 structuring element; use function `imopen` for other structuring elements.
`remove`	Remove "interior" pixels (foreground pixels that have no background neighbors).
`shrink`	Shrink objects with no holes to points; shrink objects with holes to rings.
`skel`	Skeletonize an image.
`spur`	Remove spur pixels.
`thicken`	Thicken objects without joining disconnected 1s.
`thin`	Thin objects without holes to minimally connected strokes; thin objects with holes to rings.
`tophat`	"Top-hat" operation using a 3 × 3 structuring element; use `imtophat` (see Section 9.6.2) for other structuring elements.

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